ALMOST-POSITIONED NUMERICAL SEMIGROUPS

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A numerical semigroup is a subset S of $\mathbb{N}$ that is closed under addition, contain 0 and has finite complement in $\mathbb{N}$.

Given a nonempty subset A of $\mathbb{N}$ we will denote by $\langle A\rangle$ the submonoid of $(\mathbb{N},+)$ generated by A, that is,

$$\langle A\rangle = \{\lambda_1 a_1 + \cdots + \lambda_n a_n \mid n \in \mathbb{N}\backslash\{0\}, a_i \in A, \lambda_i \in \mathbb{N} \text{ for all } i \in \{1,\ldots,n\}\}.$$

It is well known that $\langle A\rangle$ is a numerical semigroup if and only if $\gcd(A) = 1$.

If S is a numerical semigroup, then $\mathrm{m}(S) = \min(S\backslash\{0\})$, $\mathrm{F}(S) = \max\{z \in \mathbb{Z} \mid z \notin S\}$ and $\mathrm{g}(S) = \mathrm{card}(\mathbb{N}\backslash S)$ (cardinality of $\mathbb{N}\backslash S$) are three important invariants called multiplicity, Frobenius number and genus of S, respectively.

A numerical semigroup S is an almost-positioned numerical semigroup (AP-semigroup, for short) if S is $\mathrm{F}(S)+\mathrm{m}(S)+1$-positioned, that is, for all $x \in \mathbb{N}\backslash S$ we have that $\mathrm{F}(s)+\mathrm{m}(S)+1-x \in S$

In this talk we give algorithmics for computing the whole set of almost-positioned numerical semigroup with fixed multiplicity and Frobenius number. Moreover, we prove Wilf's conjecture for this type of numerical semigroups.

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